TOPIC → To find the General solution of linear Differential equation of the form when $f(D)y = e^{ax}$, when $f(a) \neq 0$

Class – B.Sc II (Maths Hons)

Date –

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Problem 1:→ solve.

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$$

Solution:→ Given equation is

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$$

Which is in symbolic form

$$(D^2 + 3D + 2)y = e^{2x}$$

Auxiliary equation

$$D^2 + 3D + 2 = 0$$

$$D^2 + 2D + D + 2 = 0$$

$$(D+2)(D+1) = 0$$

$$D = -1, -2$$

$$\therefore C.F = c_1 e^{-x} + c_2 e^{-2x}$$

$$f(D) = D^2 + 3D + 2$$

particular integral, is Given by

$$y = \frac{1}{f(D)} e^{2x}$$

$f(2) = 2^2 + 3.2 + 2 = 12 \neq 0.$

$$y = \frac{1}{f(2)} e^{2x} = \frac{1}{12} e^{2x}$$

$\therefore$ <u>General solution</u>

$$y = C.F + P.I$$

$$= c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{12} e^{2x}$$

2. $$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + k^2 y = e^x$$

<u>Given Equation</u> in Symbolic form

$$(D^2 - 2kD + k^2)y = e^x$$

$$f(D) = D^2 - 2kD + k^2$$

$$f(1) = 1 - 2k + k^2 = k^2 - 2k + 1 \neq 0.$$

Now for PI: $y = \frac{1}{f(1)} e^x = \frac{1}{k^2-2k+1} e^x = \frac{1}{(1-k)^2} e^x$

for C.F

Auxiliary Equation

$$D^2 - 2kD + k^2 = 0$$

$$(D-k)^2 = 0$$

$$D = k, k,$$

$\therefore$ $CF = (c_1 + c_2 x) e^{kx}$

$\therefore$ General solution $= C.F + PI = (c_1 + c_2 x) e^{kx} + \frac{1}{(1-k)^2} e^x$

Problem 3.

~~Problem 3~~ $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{3x}$

Solution → Given Equation can be written in Symbolic Form

$(D^2 - 4D + 4)y = e^{3x}$

Here $f(D) = D^2 - 4D + 4$

$f(3) = 9 - 12 + 4 = 1 \neq 0$

$\therefore PI = \frac{1}{f(3)} e^{3x}$

$= e^{3x}$

for C.F

Auxiliary Equation

$f(D) = 0.$

$D^2 - 4D + 4 = 0$

$(D-2)^2 = 0$

$D = 2, 2.$

$\therefore C.F = (C_1 + C_2x)e^{2x}$

$\therefore$ General Solution

$y = C.F + P.I$

$= (C_1 + C_2x)e^{2x} + e^{3x}$